

## Modeling and Projection of Some Fertility Parameters of Malaysian Population

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### ABSTRACT

*Fertility plays a major role in changing population dynamics of a country and thus understanding future fertility is important for predicting population scenario in future. The aim of the study is to develop predictive models of fertility schedule for Malaysia at different points of time. To fulfil the aim, the study uses secondary sources of data on fertility schedule such as crude birth rate (CBR) and total fertility rate (TFR) of Malaysia which are readily available from Statistics Office of the Government of Malaysia. Polynomial models were fitted to fertility schedule of Malaysia by the Gauss-Newton method using the software STATISTICA and the validity of the model was examined by cross-validity prediction power (CVPP). Thereafter, projections of fertility schedule, CBR and TFR of Malaysia were completed by employing exponential growth rate method for the period of 2013-2040. It was observed that fertility schedule of Malaysia at different points of time follow cubic polynomial models but for 2012 it follows bi-quadratic polynomial model. Using CVPP criterion and  $R^2$ , it was found that these models are well. Moreover, it was investigated that all fertility parameters used in this study were showing decreasing trend over time and these were projected up to 2040 and these projections were also displaying downward trend in accordance with time.*

**Keywords:** *Fertility Schedule, Crude Birth Rate, Total Fertility Rate, Malaysia, Polynomial Model, Cross-Validity Prediction Power, Exponential Growth Rate Method*

### 1. INTRODUCTION

In population studies, relationships among various demographic phenomena are investigated by using mathematical models which can be useful for demographers in

understanding the various important variables. Modeling is very essential for the estimation of population projections, estimations as well as any real phenomenon of the world. Census is the principal and comprehensive source of population related data. Most countries carry out their population census once in every 10 years. Similarly, in Malaysia, population and housing census is carried out once in every 10 years and the last census was conducted in 2010. Since census is only conducted once in ten years, it is important to have population projections, and estimations as well as projections of fertility and mortality parameters for future planning. These projections and estimates are useful to Government as well as NGOs and any other development partners who may require such information.

Fertility has a major impact on the size and the age structure of population in any area. Fertility refers to actual reproductivity of a woman or group of women. There are various measures of fertility (called fertility parameters) such as Crude Birth Rate (CBR), Fertility Schedule, Total Fertility Rate (TFR), Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR). However, in this study, we will only consider CBR, Fertility Schedule and TFR due to availability of the data.

There are basically two methods of carrying out population projections namely the Cohort Component Method (CCM) and the mathematical approach. In developed countries, the CCM is one of the widely used methods of population projections because of the availability of sufficient data (like birth, death and migration). With insufficient data, perhaps a mathematical approach might be more suitable in order to make projections. Projections using exponential growth rate method had been used by Islam (2007 & 2012), Islam and Beg (2009 & 2010) and Islam and Hoque (2015). Keilman and Pham (2000) fitted gamma curve to age pattern of fertility schedule in Norway. Islam and Ali (2004) investigated fertility schedule of rural areas of Bangladesh which follows a cubic polynomial model. Islam (2005) reported that marital fertility rates of urban areas of Bangladesh followed a third degree polynomial model. Therefore, in this study, we investigate what types of models are to be appropriate for the fertility schedule of Malaysian population. The specific objectives of this study are considered as follows.

- i) To build up some mathematical models to fertility schedule of Malaysia and to use Cross- Validity Prediction Power (CVPP) to check the validity of these fitted models.
- ii) To project fertility schedule, CBR and TFR of Malaysian population employing exponential growth rate method during 2013-2040.

In section 2, methodology is discussed while in section 3 the results are presented and discussed. Finally, in section 4, the conclusions are drawn.

## 2. METHODOLOGY

In this study, the secondary data of fertility schedule, CBR and TFR of Malaysia have been taken from the website of Department of Statistics, Govt. of Malaysia (<https://www.statistics.gov.my>).

**Table 1.** Observed Fertility Schedule, TFR and CBR of Malaysian Population at Different Points of Time during 1960-2012.

Age	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010	2012
15-19	114	80	58	53	43	35	29	23	17	13	14	13
20-24	306	261	226	198	183	167	139	131	99	73	58	53
25-29	301	302	262	240	237	239	209	203	187	147	128	126
30-34	241	241	214	183	180	193	172	165	162	134	131	132
35-39	160	155	142	122	106	115	104	96	95	77	73	75
40-44	68	67	57	49	42	41	40	34	32	25	21	21
45-49	19	20	18	10	7	6	5	4	4	3	2	2
<b>TFR</b>	6.045	5.630	4.885	4.275	3.990	3.980	3.490	3.278	2.983	2.355	2.136	2.118
<b>CBR</b>				30.7	30.6	31.5	27.9	26.1	23.4	18.5	17.2	17.2

**Source:** Department of Statistics, Government of Malaysia

Using the scattered plot of fertility schedule of Malaysia by ages (Fig.1-Fig.4), it appears that fertility schedule are showing nonlinear pattern with respect to ages in years. Therefore, an  $n$ th degree polynomial model is considered in the case of Malaysia and the structure of the polynomial model (Montgomery and Peck, 1982) is

$$y = a_0 + \sum_{i=1}^n a_i x^i + u$$

where  $x$  is the mid value of age group,  $y$  is fertility schedule of Malaysia,  $a_0$  is the constant;  $a_i$  is the coefficient of  $x^i$  ( $i=1, 2, 3, \dots, n$ ) and  $u$  is the error term of the model. Here, we have also to select a suitable  $n$  for which the error sum of square is minimized. These models have been fitted by Gauss-Newton Method (Fletcher, 1987) using the statistical software STATISTICA.

To examine how much those models of fertility schedule of Malaysia are stable over the population, the cross validity prediction power (CVPP) denoted by  $\rho_{cv}^2$  is applied. The CVPP is defined as

$$\rho_{cv}^2 = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)} (1 - R^2)$$

Where,  $n$  is the number of observations,  $k$  is the number of explanatory variables in the model and the cross-validated  $R$  is the correlation between observed and predicted values of the dependent variable (Stevens, 1996). It is noted that this technique is also used as model validation technique (Islam & Hossain, 2013; Hossain & Islam, 2013; Islam *et al.*, 2013, Islam *et al.*, 2020).

To verify the measure of overall significance of the model as well as the significance of  $R^2$ , the F-test is applied to the model. The  $F$ -statistical test is given as follows:

$$F = \frac{R^2 / (k-1)}{(1-R^2) / (n-k)} \quad \text{with } (k-1, n-k) \text{ degrees of freedom (d.f.)}$$

where  $k$  is the number of parameters to be estimated,  $n$  is the number of cases and  $R^2$  is the coefficient of determination of the model (Gujarati, 1998).

For the second phase procedure, i.e., for the projection of fertility schedule, CBR and TFR of Malaysia, exponential growth rate method (Shryock *et al.*, 1975) is considered in this study and the mathematical formulation of this method is given by

$$P_{t_2}^{a-(a+4)} = P_{t_1}^{a-(a+4)} \exp \left\{ r^{a-(a+4)} (t_2 - t_1) \right\} \quad \dots (1),$$

where,  $P_{t_1}^{a-(a+4)}$  is the initial population at time  $t_1$  (2000) in the age group  $a$  to  $(a+4)$ ,

$P_{t_2}^{a-(a+4)}$  = the terminal population at time  $t_2$  (2012) in the age group  $a$  to  $(a+4)$ ,

$r^{a-(a+4)}$  is the annual growth rate during the interval in the age group  $a$  to  $(a+4)$  and  $(t_2 - t_1)$  is the time interval between two time points.

The estimation of annual growth rate ( $r^{a-(a+4)}$ ) is computed for different age groups from (1) as follows.

$$r^{a-(a+4)} = \frac{1}{(t_2 - t_1)} \ln \left( \frac{P_{t_2}^{a-(a+4)}}{P_{t_1}^{a-(a+4)}} \right) \dots \quad (2)$$

In this study, the years 2000 and 2012 are considered as the initial and the terminal points respectively in estimating the growth rates using equation (2). This process is carried out successively 28 times for fertility schedule, CBR and TFR of Malaysia separately.

### 3. RESULTS AND DISCUSSION

The fitted polynomial model for the years 1960, 1980, 2000 and 2012 are as follows.

$$\text{For 1960, } \hat{y} = -2191.35 + 234.56x - 6.95x^2 + 0.06x^3 \dots \quad (i)$$

with  $R^2$  is 0.98823 and  $\rho_{cv}^2 = 0.932735$

$$\text{For 1980, } \hat{y} = -1949.68 + 199.68x - 5.83x^2 + 0.05x^3 \dots \quad (ii)$$

giving  $R^2 = 0.99441$  and  $\rho_{cv}^2$  is 0.968063

$$\text{For 2000, } \hat{y} = -1415.06 + 138.23x - 3.82x^2 + 0.03x^3 \dots \quad (iii)$$

with  $R^2$  is 0.94795 and  $\rho_{cv}^2 = 0.702571$

$$\text{For 2012, } \hat{y} = 1895.296 - 304.196x + 17.15575x^2 - 0.39370x^3 + 0.003152x^4 \dots \quad (iv)$$

providing  $R^2 = 0.98640$  and  $\rho_{cv}^2$  is 0.766857

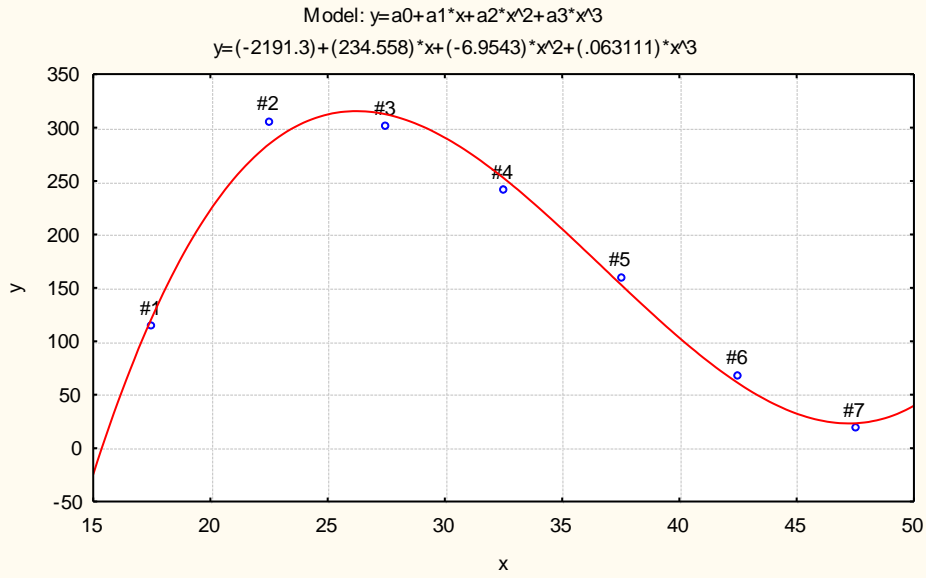
In Table 2, a summary of the fitted models is given.

Table 2. The Results of CVPP and Information on Model Fittings

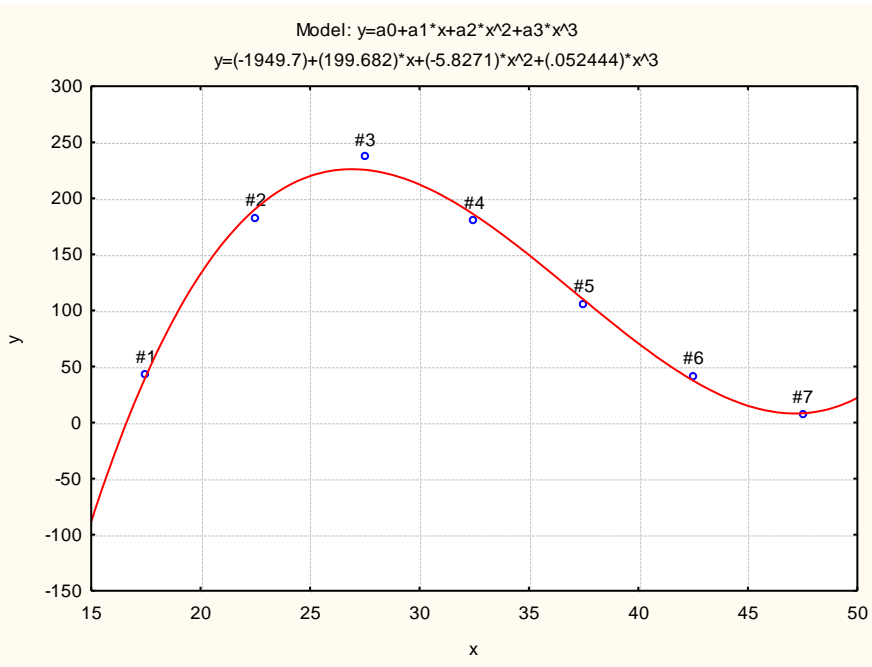
Models	De gree	Estimated Parameters	P-value	$R^2$	$\rho_{cv}^2$	Cal. F	Tab. F at 1%/5% level
<b>(i) for 1960</b>	3			0.98823	0.932735	235.55	29.46 with (3, 3) d.f. at 1% level
		$a_0$	0.004415				
		$a_1$	0.003861				
		$a_2$	0.004894				
		$a_3$	0.006881				
<b>(ii) for 1980</b>	3			0.99441	0.968063	398.93	29.46 with (3, 3) d.f. at 1% level
		$a_0$	0.00098				
		$a_1$	0.000974				
		$a_2$	0.001301				
		$a_3$	0.001892				
<b>(iii) for 2000</b>	3			0.94795	0.702571	37.53	29.46 with (3, 3) d.f. at 1% level
		$a_0$	0.032185				
		$a_1$	0.036135				
		$a_2$	0.052639				
		$a_3$	0.081373				
<b>(iv) for 2012</b>	4			0.98640	0.766857	73.29	9.12 with (4, 2) d.f. at 5% level
		$a_0$	0.120				
		$a_1$	0.093				
		$a_2$	0.07412				
		$a_3$	0.06432				
	$a_4$	0.05966					

The estimated CVPP corresponding to their  $R^2$  is shown in Table 2. From this table it is seen that all the fitted models are highly cross-validated with their large proportion of variance explained. Moreover, from this table, it is shown that the parameters of the fitted models are statistically significant. Furthermore, from the last column of Table 2, it is found that these models and their corresponding  $R^2$  are highly statistically significant in accordance with F-statistics.

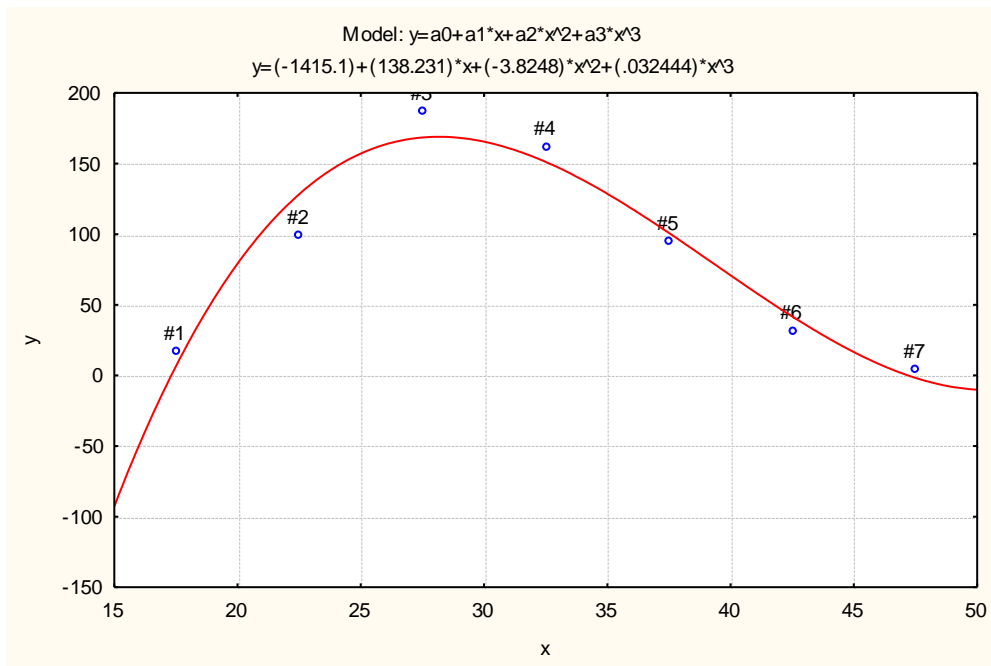
The fitted curves are shown in Fig.1 - Fig. 4.



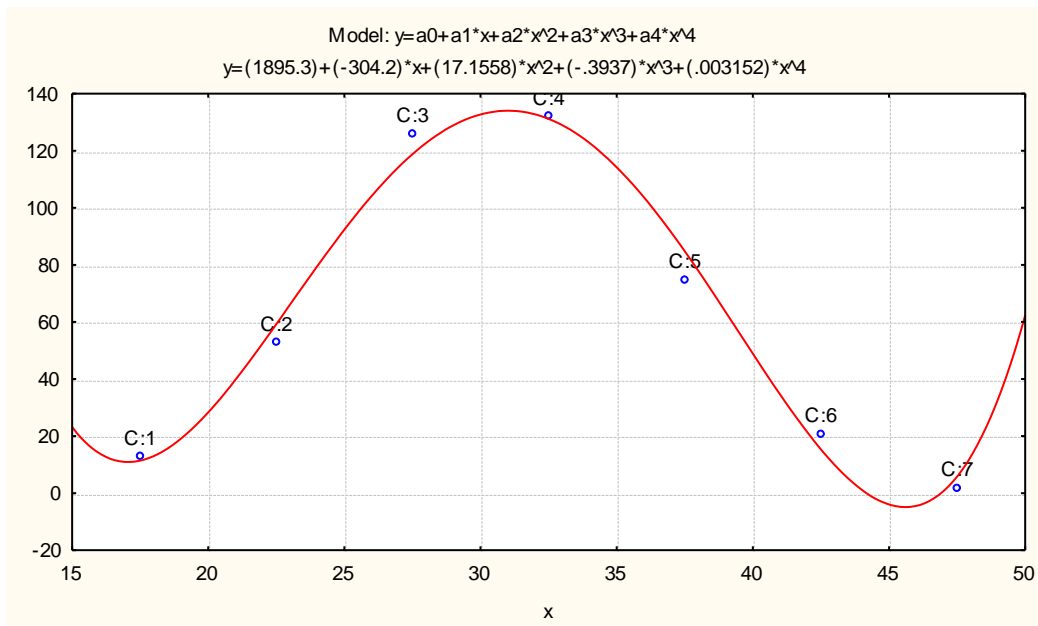
**Fig. 1** Observed and Fitted Fertility Schedule of Malaysia in 1960. X: Age Group in Years and Y: Fertility Schedule.



**Fig. 2** Observed and Fitted Fertility Schedule of Malaysia in 1980. X: Age Group in Years and Y: Fertility Schedule.



**Fig. 3** Observed and Fitted Fertility Schedule of Malaysia in 2000. X: Age Group in Years and Y: Fertility Schedule.



**Fig. 4** Observed and Fitted Fertility Schedule of Malaysia in 2012. X: Age Group in Years and Y: Fertility Schedule.



Note that we also fitted other models like log-linear, semi-log-linear, logistic, etc but these fitted models did not have good  $R^2$  values. As such, the fitting of these models are not included in this paper.

To see the trend and pattern of fertility schedule of Malaysia, observed fertility schedule of Malaysia are plotted from years for 1960 to 2012 in Fig.5. We noticed from Fig.5 that the shapes of the curves are almost like an inverted “V” for each year. Furthermore, it is observed that the peck in 1960 is in the age group 20-24 years but the peck shifts to the age group 25-29 years during 1965-2005. However, it is interesting to note that for 2010 and 2012 the peck has further shifted to the age group 30-34. This has happened due to late marriages, use of contraception, pursuing higher education, searching better employment, etc. Then, projection of fertility schedule of Malaysia using data of 2000 and 2012 by applying exponential growth rate method are estimated and presented in Table 3. Which reveals that projected fertility schedule are exhibiting downward trend at each age group and showing as usual pattern. For clear understanding the situation of the fertility schedule of Malaysia, some observed and projection fertility schedule at different points of time are displayed in the figure named Fig.6. Fertility schedule in terms of age group by time are presented in Fig. 7. Moreover, in the case of CBR and TFR, both of them are projected using same method presented in the same table and they are showing gradually decreasing trend during the observed period as well as projected period of 2013-2040.

**Table 3.** Projection of Fertility Schedule, Crude Birth Rate (CBR) and Total Fertility Rate (TFR) of Malaysian Population during 2013-2040.

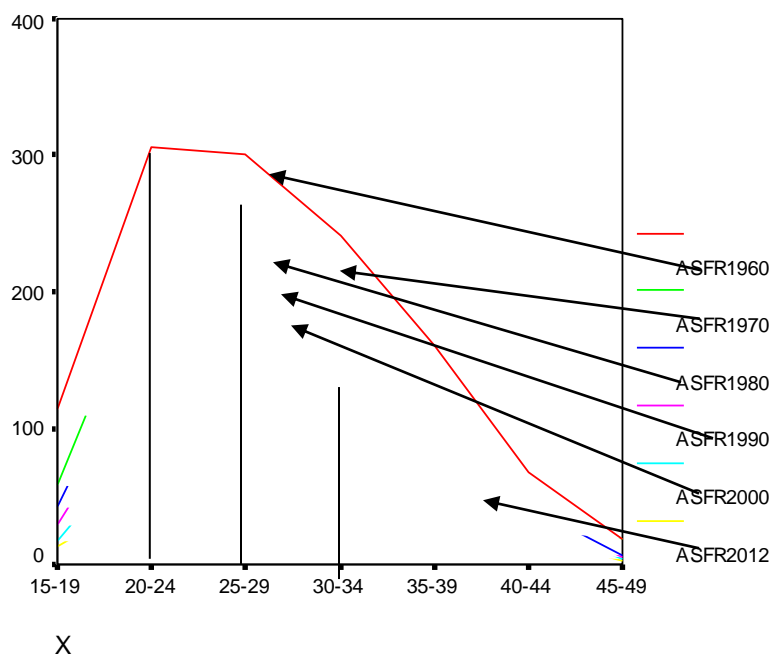
Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
15-19	13	12	12	12	12	11	11	11	11	10
20-24	50	48	45	43	41	39	37	35	33	31
25-29	122	118	114	110	107	103	100	97	94	91
30-34	130	128	125	123	121	119	117	115	113	111
35-39	74	72	71	69	68	67	65	64	63	62
40-44	20	20	19	18	18	17	16	16	15	15
45-49	2	2	2	2	1	1	1	1	1	1
<b>CBR</b>	16.764	16.340	15.926	15.523	15.130	14.746	14.373	14.009	13.654	13.308
<b>TFR</b>	2.052	1.996	1.942	1.889	1.838	1.789	1.741	1.695	1.650	1.607

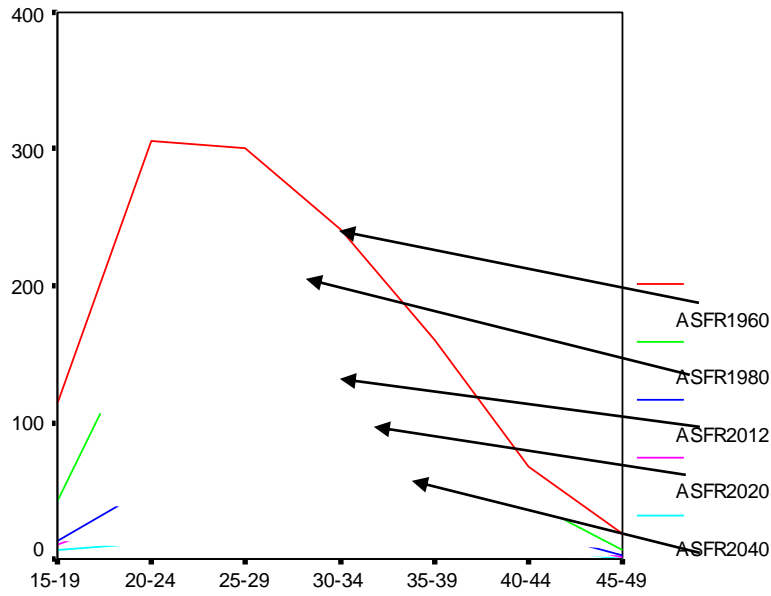
Age	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
15-19	10	10	10	10	9	9	9	9	9	8
20-24	30	28	27	26	24	23	22	21	20	19

25-29	88	85	82	79	77	74	72	70	67	65
30-34	109	108	106	104	102	100	99	97	95	94
35-39	60	59	58	57	56	55	54	53	52	51
40-44	14	14	13	13	12	12	12	11	11	10
45-49	1	1	1	1	1	1	1	1	1	1
<b>CBR</b>	12.971	12.643	12.323	12.010	11.706	11.410	11.121	10.839	10.565	10.297
<b>TFR</b>	1.565	1.524	1.484	1.446	1.409	1.373	1.338	1.304	1.271	1.239

Age	2033	2034	2035	2036	2037	2038	2039	2040
15-19	8	8	8	8	7	7	7	7
20-24	18	17	16	15	14	14	13	12
25-29	63	61	59	57	55	54	52	50
30-34	92	91	89	88	86	85	83	82
35-39	50	49	48	47	46	45	44	43
40-44	10	10	9	9	9	8	8	8
45-49	1	1	1	1	0	0	0	0
<b>CBR</b>	10.036	9.782	9.534	9.293	9.058	8.828	8.605	8.387
<b>TFR</b>	1.208	1.177	1.148	1.120	1.092	1.065	1.039	1.014

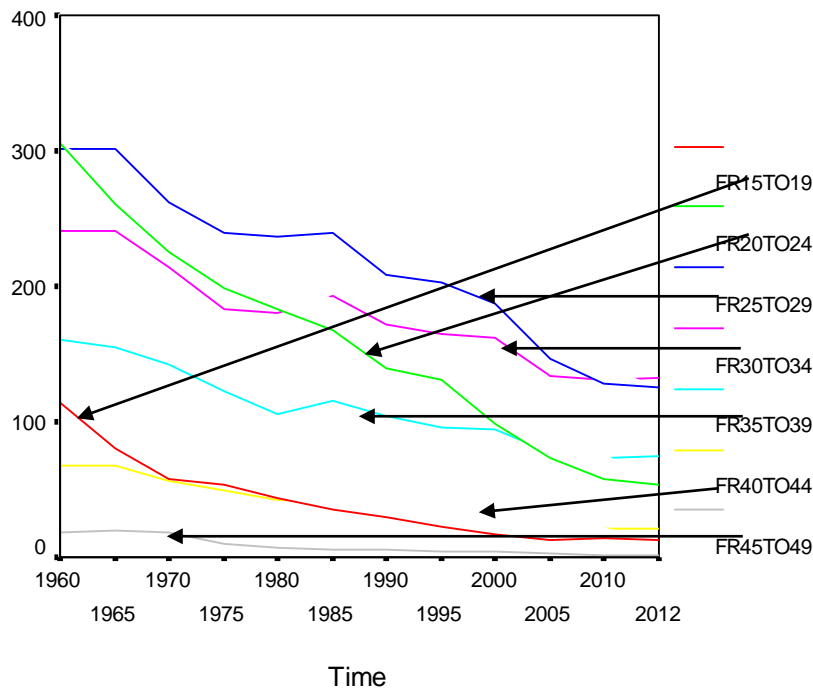


**Fig. 5** Observed Fertility Schedule of Malaysia at Different Points of Time. X: Age Group in Years and Y: Fertility Schedule.



X

**Fig. 6** Some Observed and Projected Fertility Schedule of Malaysia at Different Points of Time. X: Age Group in Years and Y: Fertility Schedule.



**Fig. 7** Observed Fertility Schedule Due to Different Age Group of Malaysia at Different Points of Time. X: Time (Year) and Y: Fertility Schedule Due to Different Age Group.

## 4. Conclusions

The objectives of this study were to build models and project fertility schedule, CBR and TFR of Malaysia. Models of fertility schedule of Malaysia at different points of time have been fitted. It is observed that fertility schedule of Malaysia in 1960, 1980 and 2000 follow the 3<sup>rd</sup> degree polynomial model but fertility schedule of Malaysia in 2012 follows bi-quadratic model. Moreover, fertility schedule, CBR and TFR of Malaysia shows a gradual decreasing trend during the observed period as well as for the projected period of 2013-2040. The decrease in these trends would perhaps lead to an ageing problem in Malaysia.

## REFERENCES

- Department of Statistics Malaysia Official Website <https://www.statistics.gov.my>, 1<sup>st</sup> June, 2015
- Fletcher, R. (1987). *Practical Methods of Optimization (2nd ed.)*. New York: John Wiley & Sons
- Gujarati, D. N. (1998). *Basic Econometrics, Third Edition*, McGraw Hill, Inc., New York.
- Hossain, M. S. & Islam, M. R. (2013). Age Specific Participation Rates of Curacao in 2011: Modeling Approach. *American Open Computational and Applied Mathematics Journal*, Vol. 1(2):08-21.
- Islam, M. R. and Ali, M.K (2004). Mathematical Modeling of Age Specific Fertility Rates and Study the Reproductivity in the Rural Area of Bangladesh During 1980-1998. *Pakistan J. of Statistics*, Vol. 20, No. 3, pp. 379-392.
- Islam, M. R. (2005). Mathematical Modeling of Age Specific Marital Fertility Rates in Urban Area of Bangladesh, *Pakistan Journal of Statistics*, Vol. 21(3): 289-295.
- Islam, M. R. (2007). Predicting Population for Male of Rural Area in Bangladesh. *Journal of Statistical Research of Iran*, Vol. 4(2): 227-238.
- Islam, M. R. and Beg, A.B.M. Rabiul Alam (2009). Modeling and Predicting Population of Dhaka District of Bangladesh, *International J. of Mathematics and Computation*, Vol. 4(9): 69-80.

- Islam, M. R. and Beg, A.B.M. Rabiul Alam (2010). Modeling and Predicting Urban Male Population of Bangladesh: Evidence from Census data, *International Journal of Mathematics And Engineering*, Vol. 1(1): 86-95.
- Islam, M. R. (2012). Modeling and Projecting Population for Muslim of Urban Area in Bangladesh, *International Journal of Probability and Statistics*, Vol. 1(1): 04-10.
- Islam, M. R. & Hossain, M. S. (2013). Mathematical Modeling of Age Specific Adult Literacy Rates of Rural Area in Bangladesh, *American Open Demography Journal*, Vol. 1(1): 01-12.
- Islam, M. R., Ali, M. K. & Islam, M. N. (2013). Construction of Life Table and Some Mathematical Models for Male Population of Bangladesh. *American Journal of Computational and Applied Mathematics*, Vol. 3(6): 269-276.
- Islam, M. R. and Hoque, M.N. (2015). Mathematical Modeling and Projecting Population of Bangladesh by Age and Sex from 2002 to 2031. *Emerging Techniques in Applied Demography*, Applied Demography Series 4, Chapter 5: 53-60.
- Islam MR., Khan MN, Hoque MN, Khan MMA (2020). Projection of Female Urban Population in Bangladesh, Population Change and Public Policy, Applied Demography Series, Vol 11. Springer**
- Keilman, N. and Pham, D.Q. (2000). Predictive Intervals for Age-Specific Fertility, *European Journal of Population*, Vol.16:41-66.
- Montgomery, D. C. and Peck, E. A. (1982). *Introduction to Linear Regression Analysis*, John Wiley and Sons, New York.
- Shryock, H. S, J. S. Siegel and Associates (1975). *The Methods and Materials of Demography*, Vol. I & II, U.S. Government Printing Office, Washington, 1973.
- Stevens, J. (1996). *Applied Multivariate Statistics for the Social Sciences*, Third Edition, Lawrence Erlbaum Associates, Inc., Publishers, New Jersey.